RG213U

```
reset():ta:=time():DIGITS:=32:c0:=299792458:Z0:=50:Z1:=50:Z2:=50:l:=
100:x:=100:Cs:=101.049872e-12:Rs:=6.56167979e-3:
```

Prozeduren: Numerische, inverse Laplace-Transformation (Talbot), für t nicht 0 eingeben

```
# Talbot suggested that the Bromwich line be deformed into a contour that begins
# and ends in the left half plane, i.e., z ? -8 at both ends.
# Due to the exponential factor the integrand decays rapidly
# on such a contour. In such situations the trapezoidal rule converge
# extraordinarily rapidly.
# For example here we compute the inverse transform of F(s) = 1/(s+1) at t = 1
#---------------------------------------------------------------------------
#Octave:
#>> pkg load symbolic
#>> syms s
#>> F=1/(s+1)
#F = (sym)
#
#   1
#  -----
#  s + 1
#
#>> error=talbot(function_handle(F),1,24)-exp(-1)
#ans =   1.6098e-015
#-----------------------------------------------------------------------------
#
# Talbot method is very powerful here we see an error of 1.61e-015
# with only 24 function evaluations
#
# Created by Fernando Damian Nieuwveldt
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# Date : 25 October 2009
#
# Reference
# L.N.Trefethen, J.A.C.Weideman, and T.Schmelzer. Talbot quadratures
# and rational approximations. BIT. Numerical Mathematics,
# 46(3):653 670, 2006.
#
#   Shift contour to the right in case there is a pole on the positive real axis : Note the contour will
#   not be optimal since it was originally devoloped for function with
#   singularities on the negative real axis
#   For example take F(s) = 1/(s-1), it has a pole at s = 1, the contour needs to be shifted with one
#   unit, i.e shift  = 1. But in the test example no shifting is necessary
```

```
Talbot:=proc(F_s, t, N)
local h,shift,ans,theta,k,z,dz;
begin
 h:=2*PI/N;
 shift:=0;
 ans:=0;
 for k from 0 to N do
```

```
      theta:=-PI+(k+1/2)*h;
      z:=shift+N/t*(0.5017*theta*cot(0.6407*theta)-
     0.6122+0.2645*I*theta);
      dz:=N/t*(-
     0.5017*0.6407*theta/sin(0.6407*theta)^2+0.5017*cot(0.6407*theta)+0
     .2645*I);
      ans:=ans+exp(z*t)*F_s(z)*dz;
    end_for:
      return (Re(h/(2*I*PI)*ans));
  end_proc:
```

Induktivitätsbelag in uH/m

- `Ls:=Z0^2*Cs:float(Ls/1e-6);`

  0.25262468

Ableitungsbelag in uS/m

- `Gs:=150*Rs*Cs/Ls:float(Gs/1e-6);`

  393.7007874

Ausbreitungsgeschwindigkeit auf der Leitung in m/s

- `vl:=1/sqrt(Ls*Cs);`

  197922071.58857163124362987812592

Verhältnis Ausbreitungsgeschw. / Lichtgeschw.

- `vl/c0;`

  0.66019696729185772659974614213251

Laufzeit für x Meter in us (2 Methoden)

- `td:=x/vl:float(td/1e-6),float(x*sqrt(Ls*Cs)/1e-6);`

  0.50524936, 0.50524936

Übertragungsfunktion der Leitung

- `gam:=sqrt((Rs+p*Ls)*(Gs+p*Cs)):`
- `/* Tp:=(Z2*cosh(gam*(l-x))+Z0*sinh(gam*(l-x)))/((Z1+Z2)*cosh(gam*l)+(Z0+Z1*Z2/Z0)*sinh(gam*l)): */`

sinh() u. cosh() umformen in e-Funktionen

- `a:=gam*(l-x):b:=gam*l:`
- `Tp1:=(Z2*(exp(a)+exp(-a))+Z0*(exp(a)-exp(-a)))/((Z1+Z2)*(exp(b)+exp(-b))+(Z0+Z1*Z2/Z0)*(exp(b)-exp(-b))):`

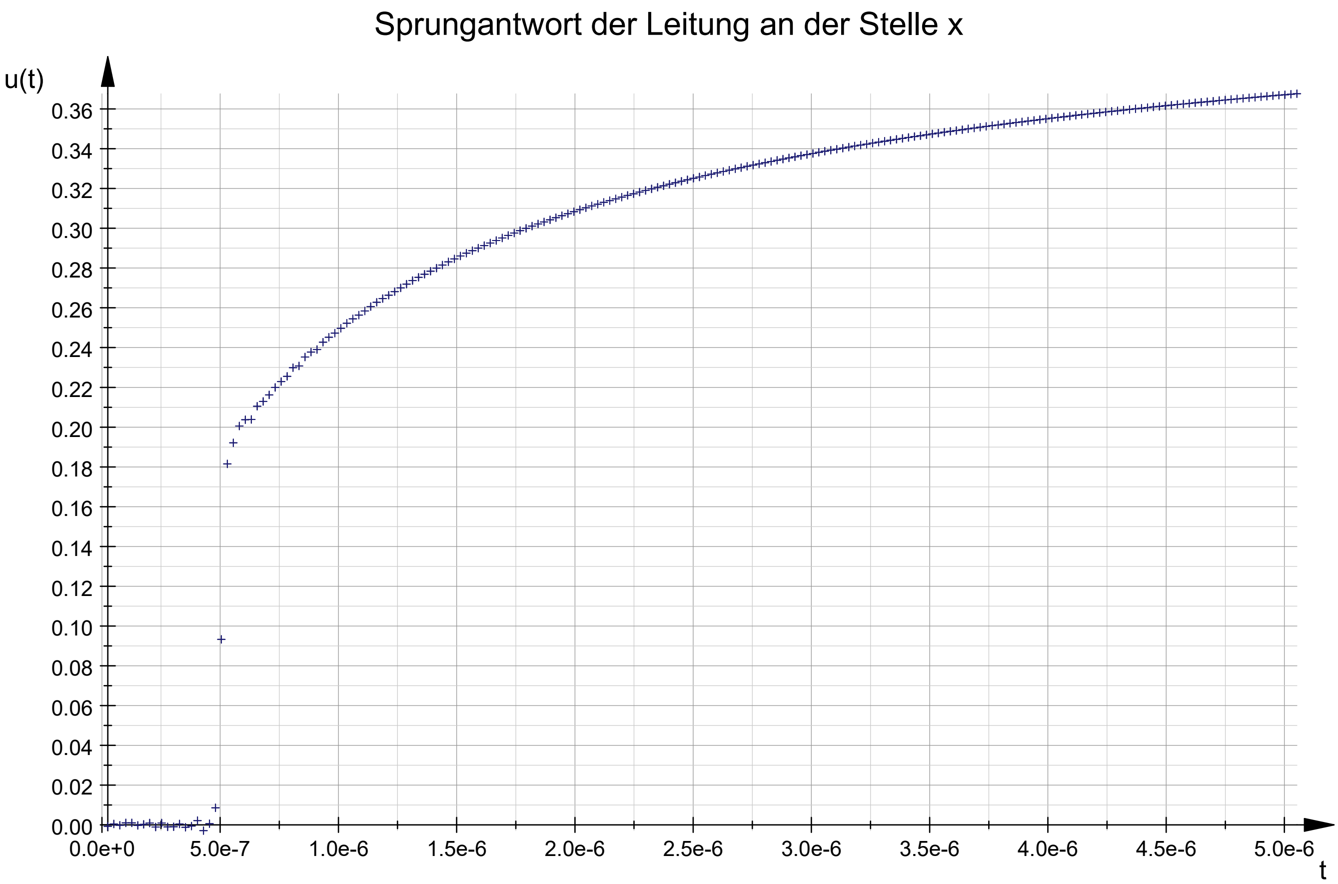
Erregung Sprungfunktion 1/p

- `lap:=(p)-->expand(1/p*Tp1);`

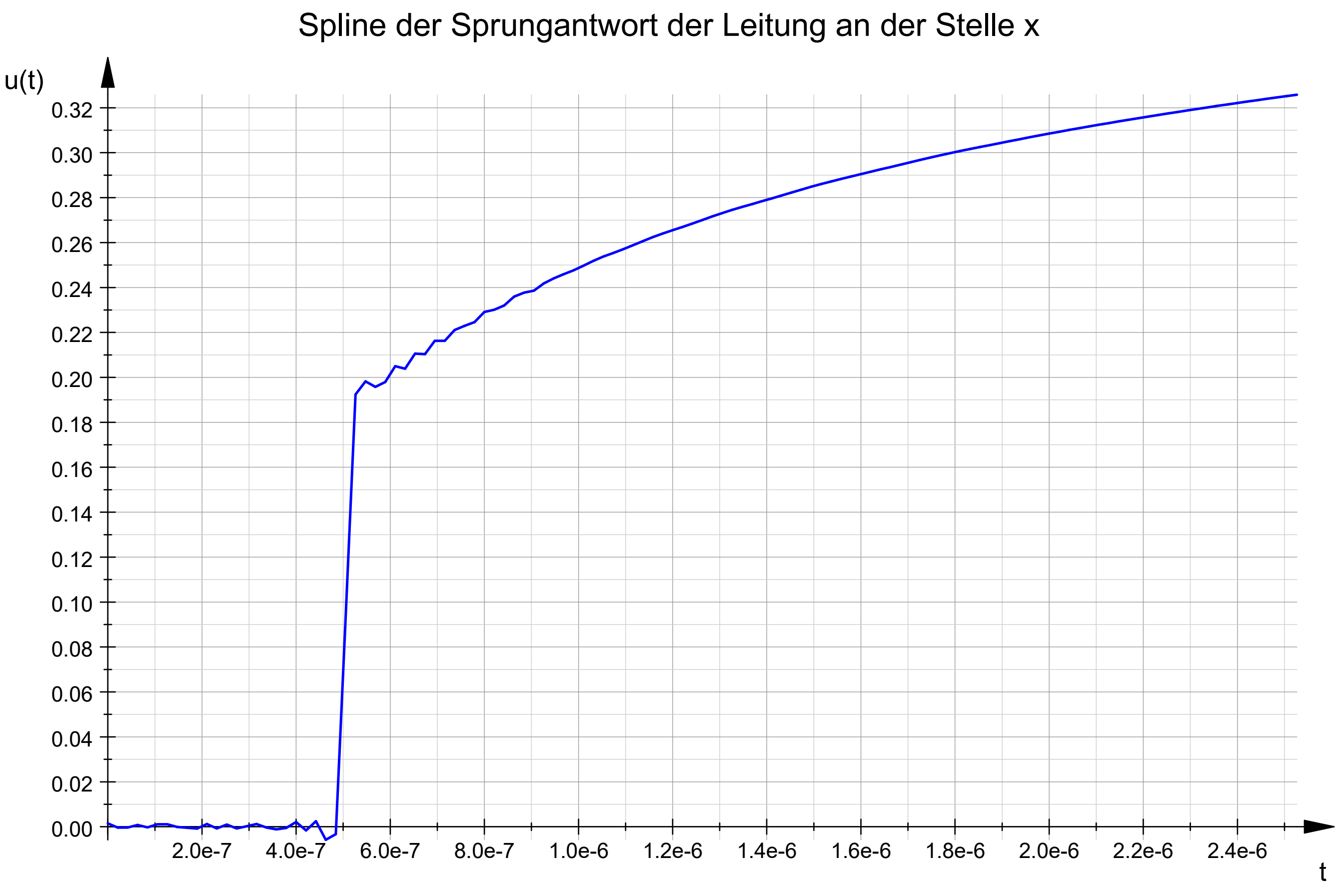
$$p \rightarrow \frac{1}{2 \cdot p \cdot \left(e^{\sqrt{0.000000000000000002552769157804096 \cdot p^2 + 0.0000000001001215923355751888 \cdot p + 0.0000025833384999896666646}}\right)^{100}}$$

Anzahl der Stützstellen u. Talbot-Iterationen

```
• M:=200:Talits:=384:
• Liste:=[[float(10*td/M*i), float(Talbot(lap,10*td/M*i,Talits))] $ i=1..M]:
• plot(plot::PointList2d(Liste, PointStyle=Crosses, PointSize=1, GridVisible=TRUE, SubgridVisible=TRUE,
                         Scaling=Unconstrained,
    AxesTitles=["t","u(t)"]), Height=120*unit::mm, Width=180*unit::mm,
    Header="Sprungantwort der Leitung an der Stelle x"):
```




```
• x:=array(0..M-1, [float(5*td/M*(i+1)) $ i=0..M-1]):
• S:=numeric::cubicSpline([x[i], float(Talbot(lap,x[i],Talits))] $ i=0..M-1, Natural):
• delete x:plot(plot::Function2d(S(x), x=0..5*td), GridVisible=TRUE, SubgridVisible=TRUE,
                         Scaling=Unconstrained,
    AxesTitles=["t","u(t)"], Height=120*unit::mm, Width=180*unit::mm,
    Header="Spline der Sprungantwort der Leitung an der Stelle x"):
```



Berechnung eines Funktionswertes

```
• S(8e-7);
```

0.22908062402378941050314896529032

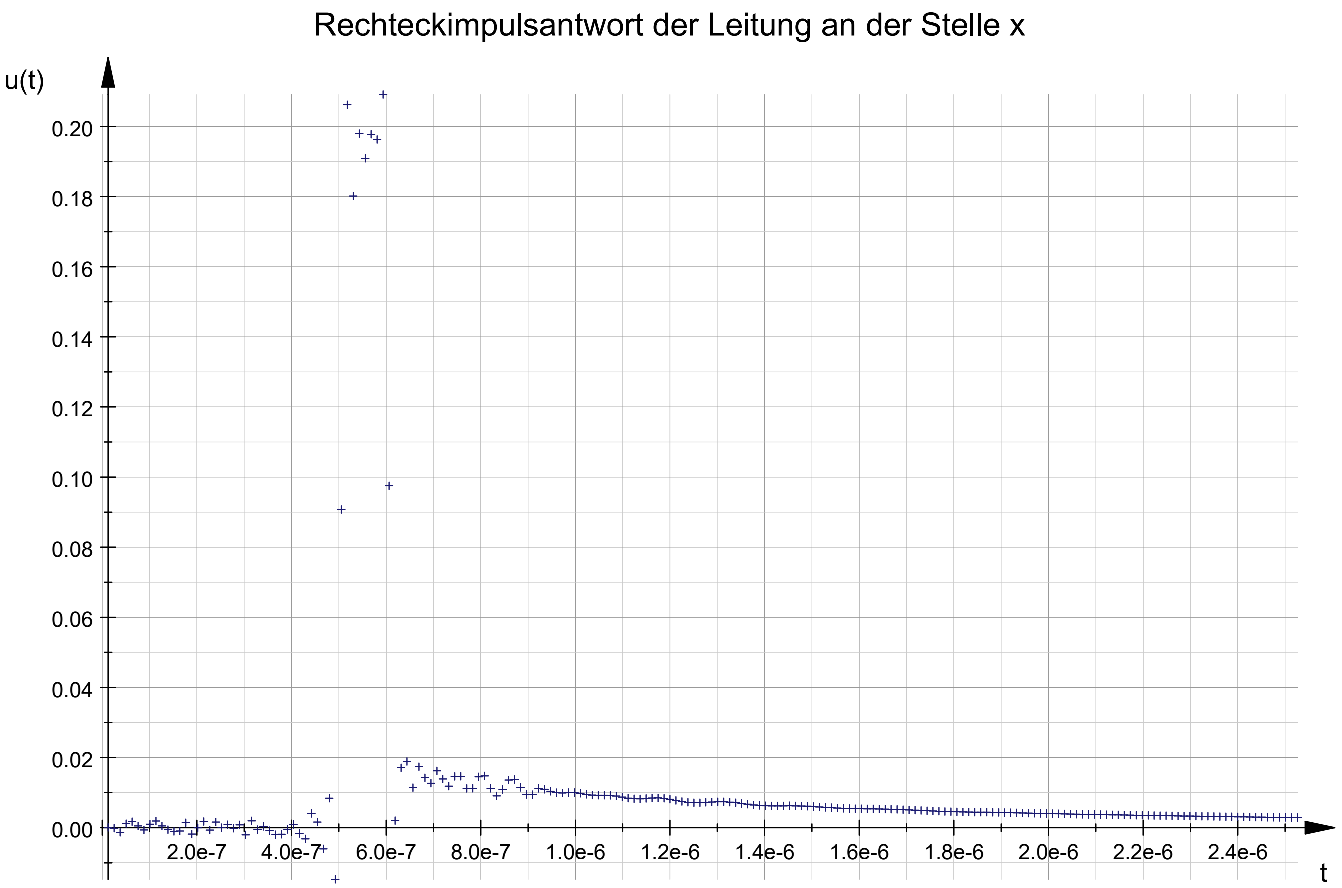
Erregung Rechteckimpuls 0.1 us

```
• Ug1:=1/p*(1-exp(-1e-7*p)):
• lap:=(p)-->expand(Ug1*Tp1);
```

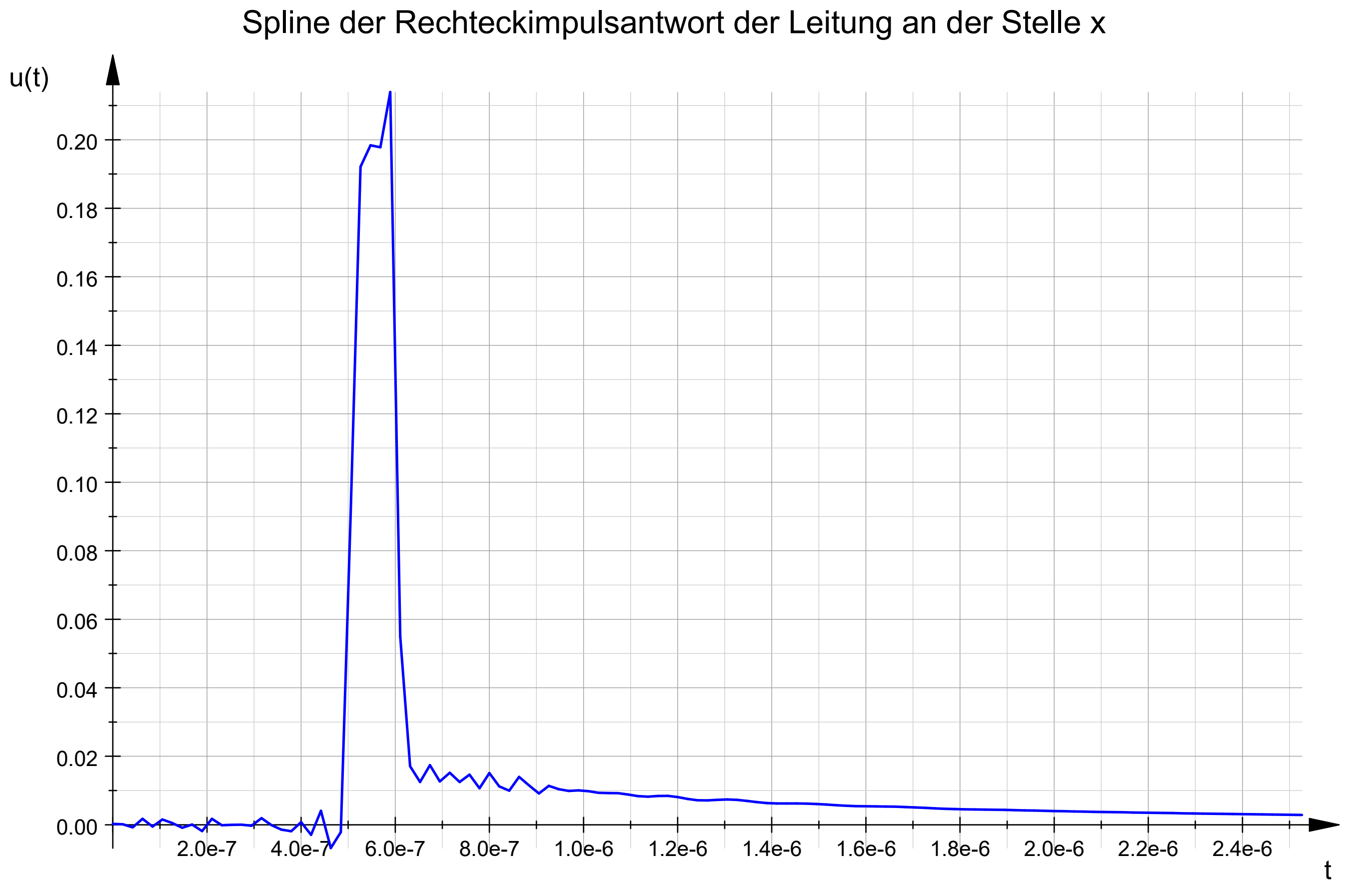
$$p \to \frac{1}{2 \cdot p \cdot \left(e^{\sqrt{0.000000000000000002552769157804096 \cdot p^2 + 0.00000000010012159233555751888 \cdot p + 0.000002583338499989666646}}\right)^{100}}$$

```
• delete Liste:Liste:=[[float(5*td/M*i),
  float(Talbot(lap,5*td/M*i,Talits))] $ i=1..M]:
• plot(plot::PointList2d(Liste, PointStyle=Crosses, PointSize=1,
  GridVisible=TRUE, SubgridVisible=TRUE,
                      Scaling=Unconstrained,
```

```
AxesTitles=["t","u(t)"]), Height=120*unit::mm, Width=180*unit::mm,
Header="Rechteckimpulsantwort der Leitung an der Stelle x"):
```



- `x:=array(0..M-1, [float(5*td/M*(i+1)) $ i=0..M-1]):`
- `S:=numeric::cubicSpline([x[i], float(Talbot(lap,x[i],Talits))] $ i=0..M-1, Natural):`
- ```
  delete x:plot(plot::Function2d(S(x), x=0..5*td), GridVisible=TRUE, SubgridVisible=TRUE,
                                  Scaling=Unconstrained,
     AxesTitles=["t","u(t)"], Height=120*unit::mm, Width=180*unit::mm,
     Header="Spline der Rechteckimpulsantwort der Leitung an der Stelle x"):
  ```
  ```



Berechnung eines Funktionswertes

- `S(8e-7);`

  0.01512283602983722583639223256606

CPU-Zeit in Sekunden und Minuten

- `float((time()-ta)/1e3);float((time()-ta)/1e3/60);`

  5092.953

  84.88255

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