Betrachtung eines Bessel-TP --- 6.Mai 2007 Ingenieurbüro Baumann, Dorsten

```
reset():ta:=time():DIGITS:=32:w:=2*PI*f:
```

die Eingangsdaten

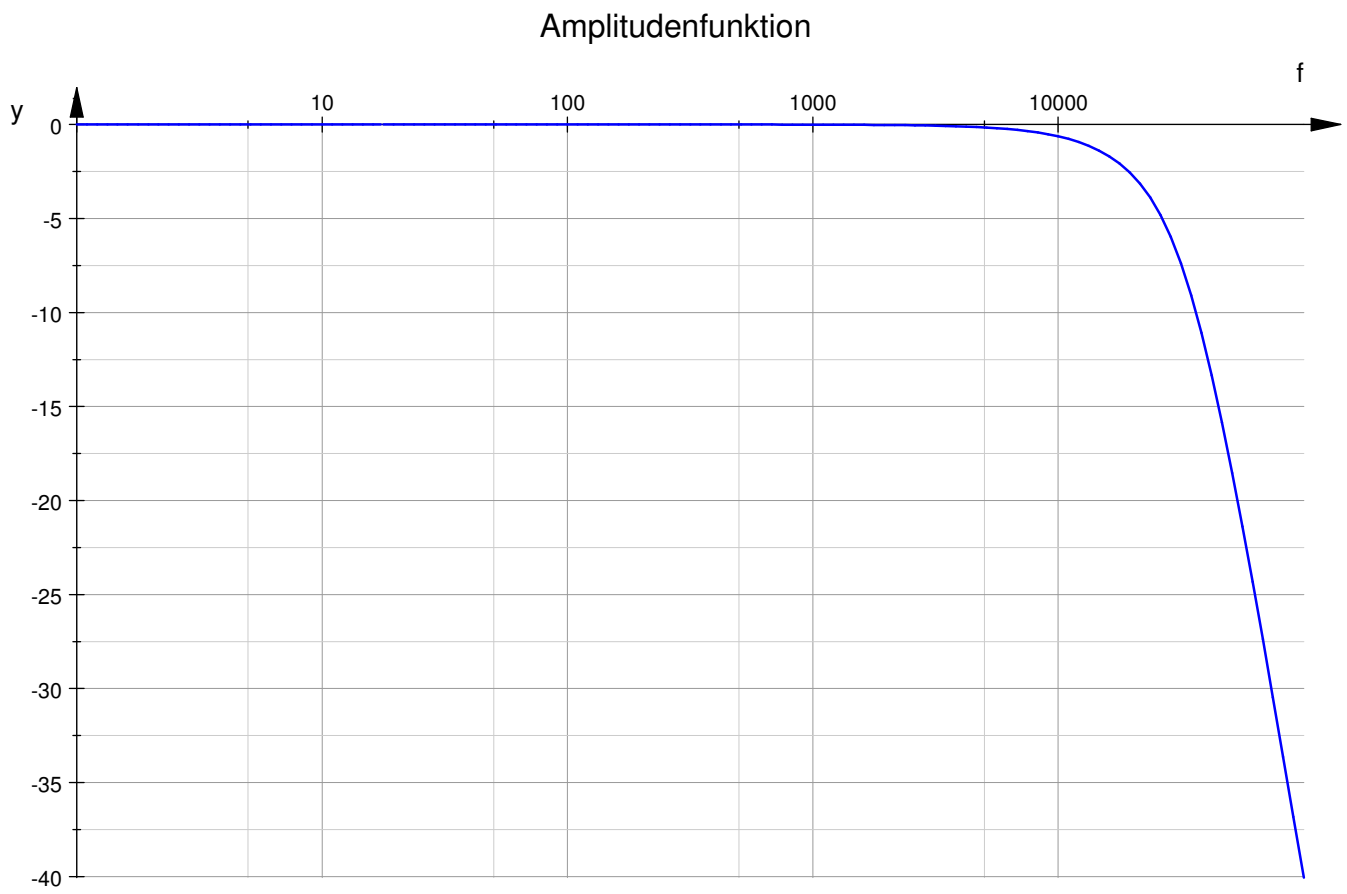
```
n:=4:fg:=10e3:ue2:=1:
```

die Berechnungen

```
wg:=2*PI*fg:
b:=[fact(2*n-(i-1))/(2^(n-(i-1))*fact(i-1)*fact(n-(i-1))) $
i=1..n+1]:b0:=b[1]:a0:=b0:
delete i:U2U0:=(f)->b0/(a0*(1+1/ue2))*a0/sum(b[i+1]*(I*w/wg)^i,
i=0..n):
U2U0dB:=(f)->20*log(10,abs(U2U0(f))):
Winkel:=(f)->180/PI*arctan(Im(U2U0(f))/Re(U2U0(f))):
Winkel:=(f)->180/PI*arg(U2U0(f)):
tg1:=(f)->-diff(Winkel(f),f)/360:
```

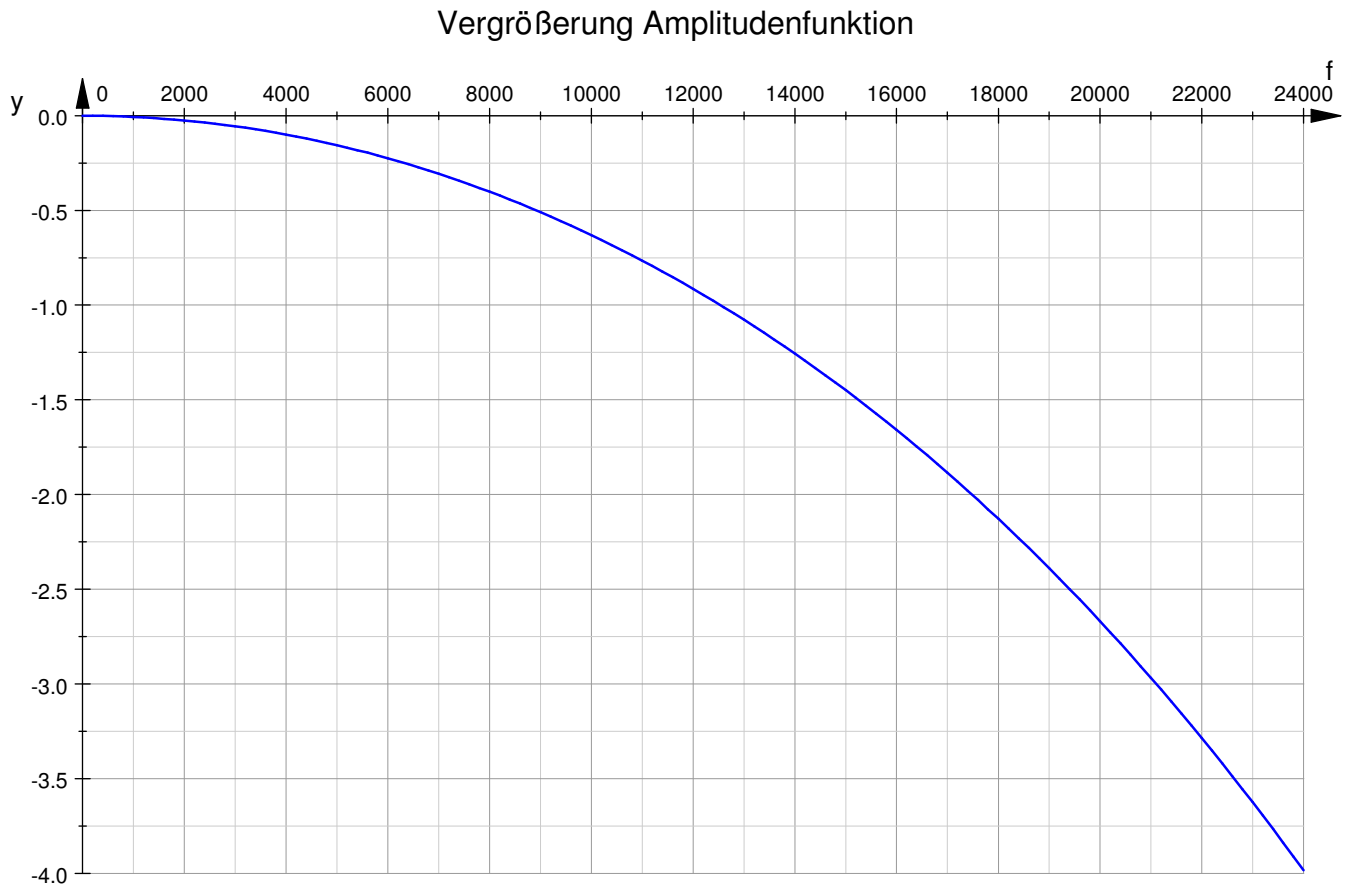
Betrag der Übertragungsfunktion, doppelt logarithmisch

```
plotfunc2d(U2U0dB(f)+6.02, f=1..10*fg, LegendVisible=FALSE,
CoordinateType=LogLin,
           GridVisible=TRUE, SubgridVisible=TRUE,
  Height=120*unit::mm, Width=180*unit::mm
  ,Header="Amplitudenfunktion"):
```



Ausschnittsvergrößerung aus dem Betrag der Übertragungsfunktion

- `plotfunc2d(U2U0dB(f)+6.02, f=0..2.4*fg, LegendVisible=FALSE, GridVisible=TRUE, SubgridVisible=TRUE, Height=120*unit::mm, Width=180*unit::mm, Header="Vergrößerung Amplitudenfunktion"):`

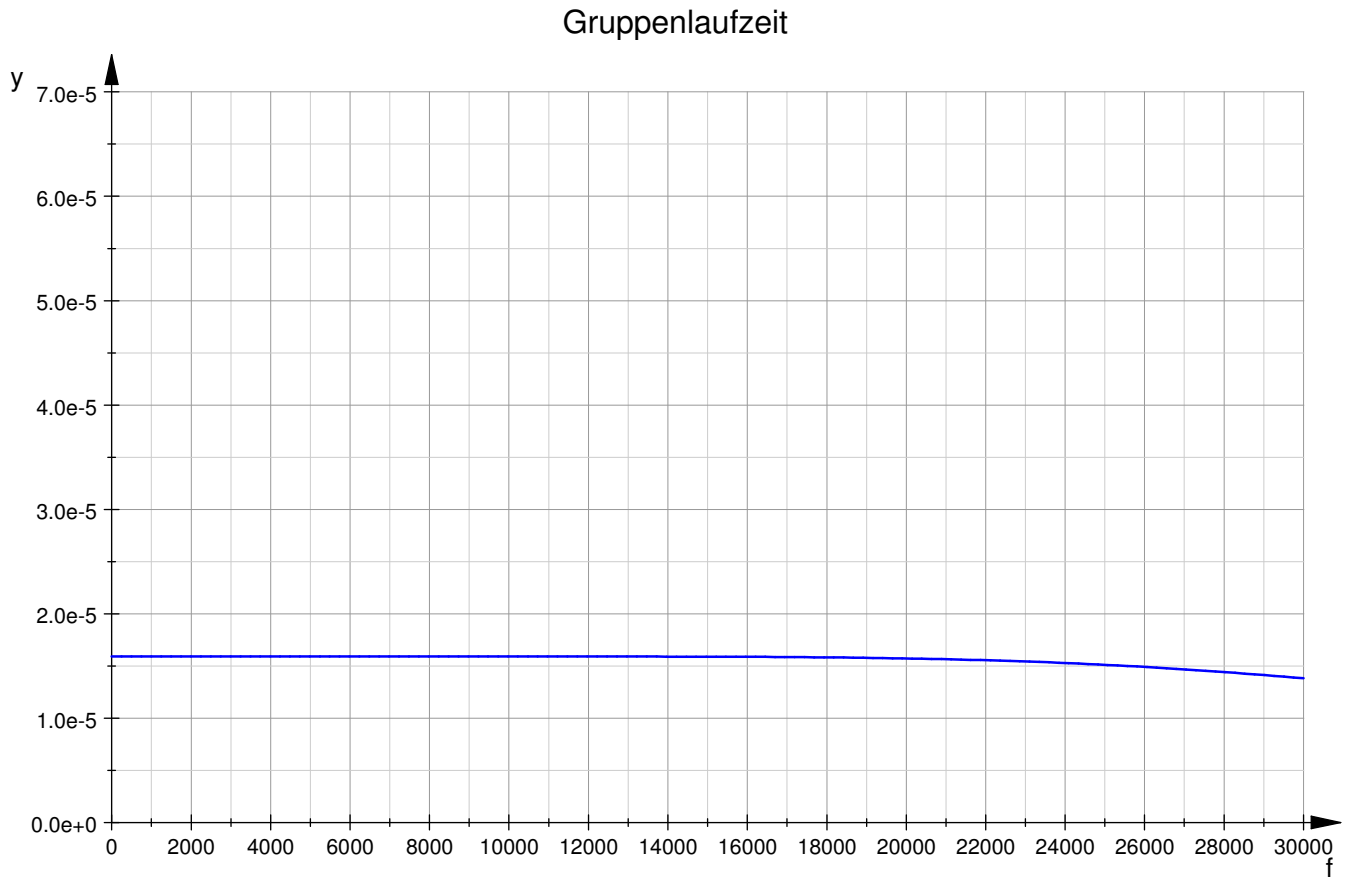


die Phasenverschiebung des Filters

- `plotfunc2d(Winkel(f), f=0..1.55*fg, LegendVisible=FALSE, GridVisible=TRUE, SubgridVisible=TRUE, Height=120*unit::mm, Width=180*unit::mm, Header="Phasenfunktion"):`

Gruppenlaufzeit aus dem differenzierten Phasenverlauf

```
plotfunc2d(tg1(f), f=0..3*fg, LegendVisible=FALSE,
           GridVisible=TRUE, SubgridVisible=TRUE,
  Height=120*unit::mm, Width=180*unit::mm, Header="Gruppenlaufzeit",
  YRange=0..7e-5):
```



das Bessel-Polynom ohne die Grunddämpfung

- `Poly:=(F)->expand(sum(b[i+1]*(F)^i, i=0..n)):b;`

$$[105,\ 105,\ 45,\ 10,\ 1]$$

- `a0/Poly(F);`

$$\frac{105}{F^4 + 10 \cdot F^3 + 45 \cdot F^2 + 105 \cdot F + 105}$$

die Grunddämpfung durch ü^2

- `b0/(a0*(1+1/ue2));`

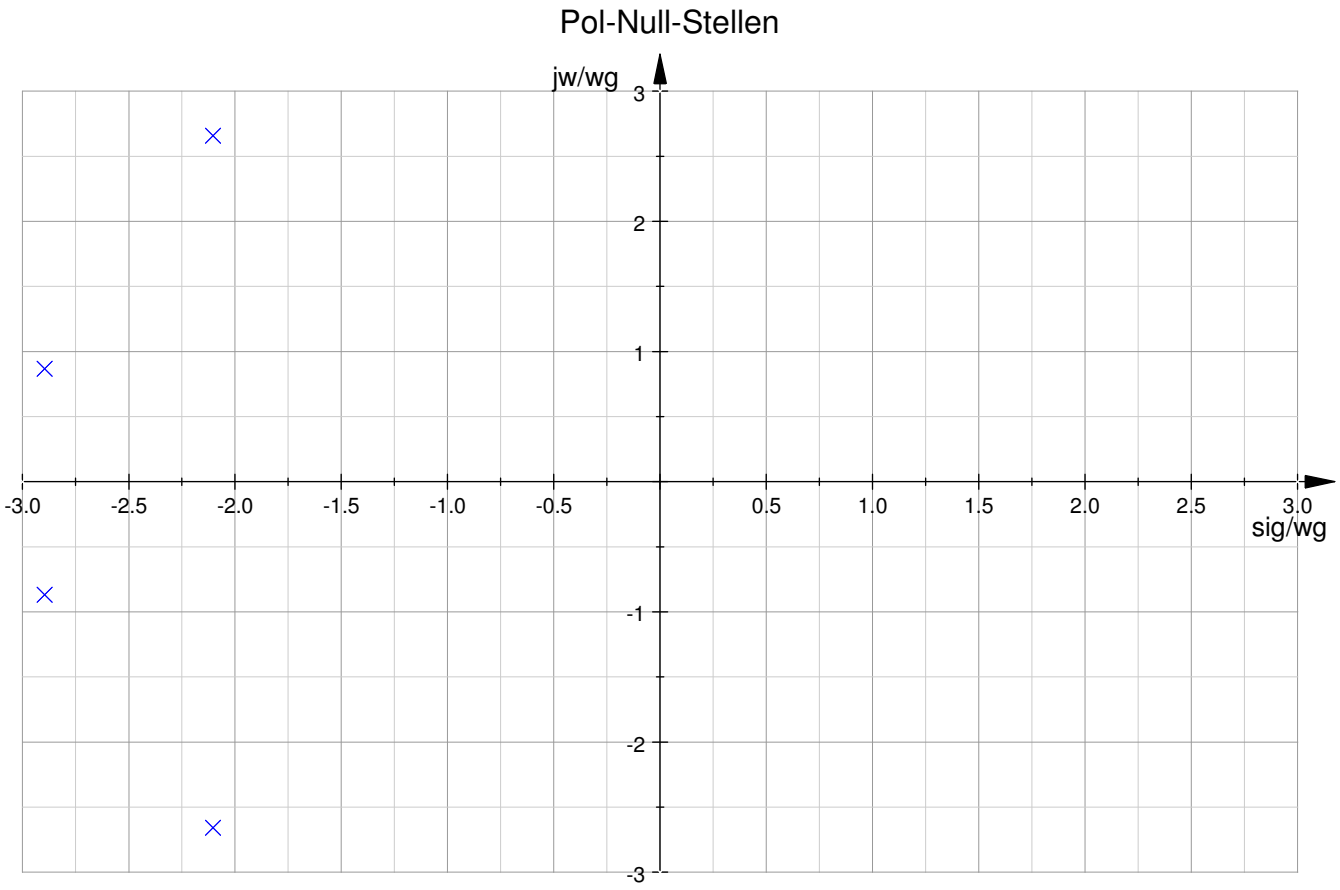
$$\frac{1}{2}$$

- `delete i:Pol:=float(solve(Poly(F)=0,F)):`
- ```
  delete Liste:for i from 1 to n do
       PolTab[i]:=op(Pol,i):
  end_for:
  ```
- `Liste:=[[Re(op(op(PolTab,i),2)),Im(op(op(PolTab,i),2)),RGB::Blue] $ i=1..n]:`
- `Liste:=Liste.[[3,0,RGB::White]].[[0,3,RGB::White]].[[-`
  ```

```
3,0,RGB::White]].[[0,-3,RGB::White]]:
```

die Lage der Pol-Nullstellen in der komplexen Ebene

- ```
  plot(plot::PointList2d(Liste, PointStyle=XCrosses, PointSize=2,
  GridVisible=TRUE, SubgridVisible=TRUE,
                         Scaling=Unconstrained,
    AxesTitles=["sig/wg","jw/wg"]), Height=120*unit::mm,
    Width=180*unit::mm, Header="Pol-Null-Stellen"):
  ```



die Pol-Nullstellen

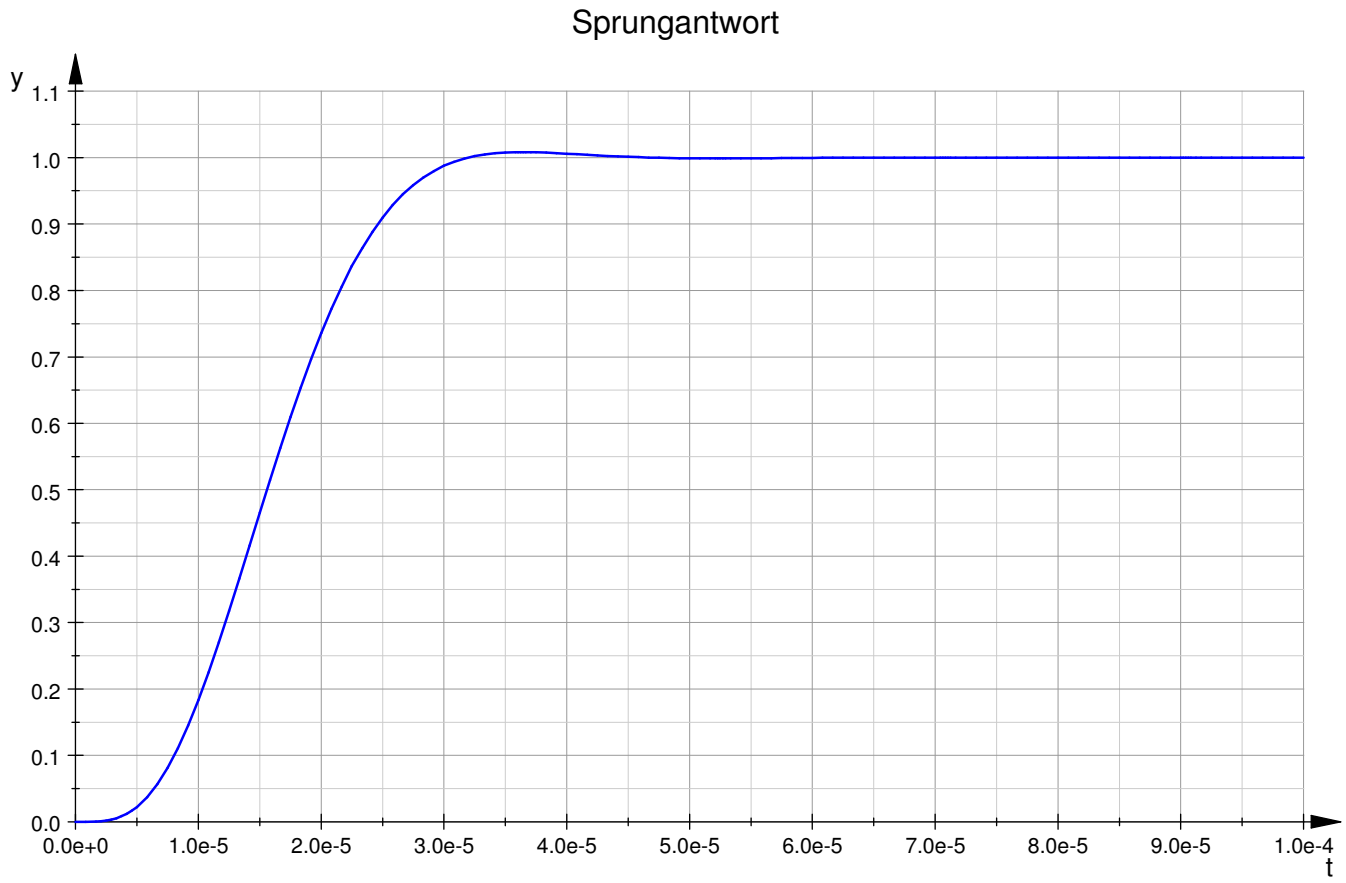
- ```
  PolTab;
  ```

$$\begin{bmatrix} 1 & = -2.103789397179627831605606074724 + 2.657418041856752716858322098631 8 \cdot i \\ 2 & = -2.103789397179627831605606074724 - 2.657418041856752716858322098631 8 \cdot i \\ 3 & = -2.896210602820372168394393925276 - 0.867234128934503751818973214920 12 \cdot i \\ 4 & = -2.896210602820372168394393925276 + 0.867234128934503751818973214920 12 \cdot i \end{bmatrix}$$

Sprungantwort des Filters mit ua(t)=invlaplace(2/p*T(p))

- ```
  prodp:=1:for i from 1 to n do
       prodp:=prodp*(p/wg-op(Pol,i))
  end_for:
  ```
- ```
  ua:=(t)-
  >Re(transform::invlaplace(b0/(a0*(1+1/ue2))*a0*2/p/prodp,p,t)):
  ```

- `plotfunc2d(ua(t), t=0..1/fg, LegendVisible=FALSE, CoordinateType=LinLin, GridVisible=TRUE, SubgridVisible=TRUE, Height=120*unit::mm, Width=180*unit::mm, Header="Sprungantwort", YRange=0..1.1):`



Suchbereich für Flankendaten

- `anf:=0:ende:=1/2/fg:`

Überschwingen in % bei t in us

- `maximum:=op(numeric::solve(diff(ua(t),t)=0,t=0..1/2/fg,RestrictedSearch),1):`
- `(ua(maximum)-1)*100;maximum/1e-6;`

0.83541995143491402793358118472718

36.354919486450169423964526030568

t0 für ua(t)=1/2 in us

- `delete t:tx:=op(numeric::solve(ua(t)=1/2,t=anf..maximum, RestrictedSearch),1):tx/1e-6;`

15.580288462336022809616470370504

die Einschwingzeit tau in us und die daraus resultierende Grenzfrequenz in kHz

- `m:=ua'(t):t:=tx:m:=float(m):delete t:yt:=t->1/2-m*(tx-t):`
- `tau:=op(solve(yt(t)=1,t),1)-op(solve(yt(t)=0,t),1):tau/1e-6;1/2/tau/1e3;`

16.958092479942069624339684619802

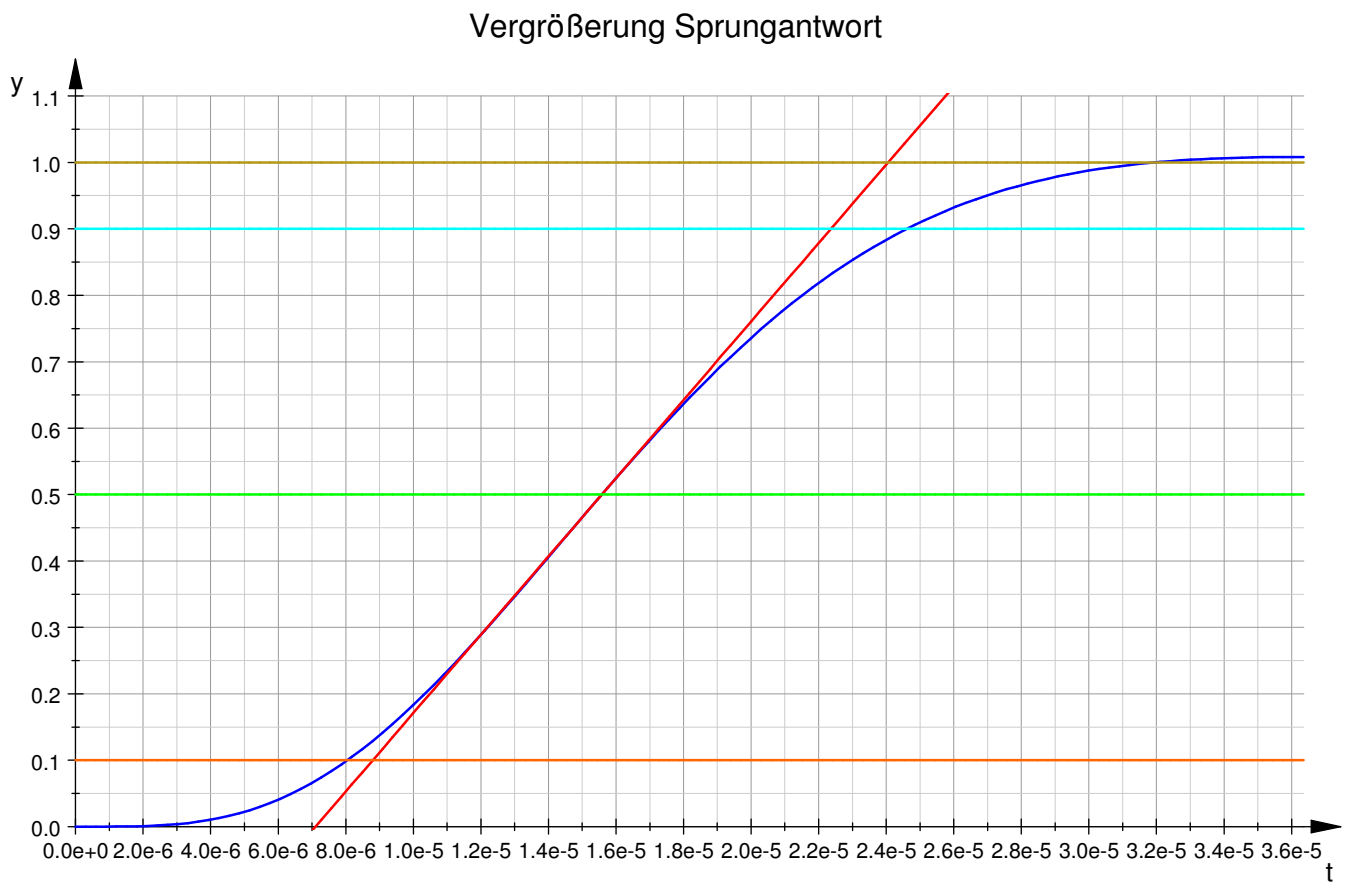
29.484448241534064635715985527527

tr, Rise-Time in us

- `tr:=op(numeric::solve(ua(t)=9/10,t=anf..ende, RestrictedSearch),1)-op(numeric::solve(ua(t)=1/10,t=anf..ende, RestrictedSearch),1):tr/1e-6;`

16.563084815194208343610079938117

- `plotfunc2d(ua(t), yt(t), 1/2, 1, 1/10, 9/10, t=0..maximum, LegendVisible=FALSE, CoordinateType=LinLin, GridVisible=TRUE, SubgridVisible=TRUE, Height=120*unit::mm, Width=180*unit::mm, Header="Vergrößerung Sprungantwort", YRange=0..1.1):`



CPU-Zeit in Sekunden und in Minuten

- `te:=time():float((te-ta)/1000);float((te-ta)/1000/60);`

  3.391

  0.05651666666666666666666666666667

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